

Article

The determination of inflection curve on a given ogive curve using the second order derivative in calculus

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Abstract: The cumulative annual number of publications continually grown was frequently applied using a literature logical growth curve and the corresponding inflection point. However, none was found using a scientific approach to verify the way for determining the inflection point. We use the second order derivative in calculus to prove the way that is suitable for use in determination of inflection point. The aims of this study include that verifying the efficacy of the exponential growth model on accumulative publications of mobile health research between 1997 and 2017 in literature. We observed that the model accuracy ($R^2 = 0.99$) is higher than the one ($R^2 = 0.98$) in literature, based on identical data. The exponential growth model can be applied to other disciplines for helping us predict the outcomes in the future. The inflection curve provides us a deeper insight into the ogive curve that represents the trajectory and trend of interest we concern about in practice.

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Introduction

In a study[1] reporting the cumulative annual number of publications continually grown from 2 to 2704 using a literature logical growth curve and the corresponding inflection point at 20 years since 1997. Although a direct fit to the equation: $y = 3913.14 / (1 + 1929.18e^{-0.39t})$, ($R^2 > 0.987$, where y is the cumulative annual number of papers and t is the number of years since 1997), the time of the inflection point of the growth curve (yielded by $t = \ln(1929.18) / 0.39 = 19.4 \approx 20$ (ie, 2016 - 1997 + 1) was not evident of literature source and from references in literature. We are motivated to verify the termination of inflection point on a given ogive curve. From which, the logical growth curve can be applied to other fields in the future.

Methods

2.1 Data source

We obtained data from the study identified the article[1] that was published in PubMed using the bibliometric analysis [2].

Two major part of this study is to verify the results reported in the study[1] which is correct and precise I nmodel- ing and determination of inflection point.

2.2 The determination of inflection curve on a given ogive curve using the second order derivative in calculus

$$f(t) = \frac{a}{1 + be^{-ct}} \quad a, b, c \text{ are none zero constants.}$$

$$\begin{aligned} f'(t) &= \left(\frac{a}{1 + be^{-ct}}\right)' = a \left(\frac{1}{1 + be^{-ct}}\right)' = -a \frac{(1 + be^{-ct})'}{(1 + be^{-ct})^2} = -a \frac{0 + (be^{-ct})'(-c)}{(1 + be^{-ct})^2} = -a \frac{be^{-ct}(-c)}{(1 + be^{-ct})^2} \\ &= [a(1 + be^{-ct})^{-2}](bce^{-ct}) \end{aligned}$$

$$\begin{aligned} f''(t) &= [f'(t)]' = \{[a(1 + be^{-ct})^{-2}](bce^{-ct})\}' = [a(1 + be^{-ct})^{-2}]'(bce^{-ct}) + [a(1 + be^{-ct})^{-2}](bce^{-ct})' \\ &= [-2a(1 + be^{-ct})^{-3} (be^{-ct})'(-c)](bce^{-ct}) + [a(1 + be^{-ct})^{-2}][(bce^{-ct})(-c)]' \\ &= [2a(1 + be^{-ct})^{-3} (be^{-ct}) c](bce^{-ct}) + [a(1 + be^{-ct})^{-2}][(bce^{-ct})(-c)]' \\ &= [2a(1 + be^{-ct})^{-3} (b^2c^2e^{-2ct})] + \{[a(1 + be^{-ct})^{-2}](-bc^2e^{-ct})\}' \\ &= [abc^2(1 + be^{-ct})^{-2} e^{-ct}] [2b(1 + be^{-ct})^{-1} e^{-ct} - 1] \\ &= [abc^2(1 + be^{-ct})^{-2} e^{-ct}] \left[\frac{2be^{-ct}}{1 + be^{-ct}} - 1 \right] = [abc^2(1 + be^{-ct})^{-2} e^{-ct}] \frac{be^{-ct} - 1}{1 + be^{-ct}} \\ &= [abc^2(1 + be^{-ct})^{-2} e^{-ct}] \frac{b - e^{ct}}{1 + be^{-ct}} = \frac{abc^2}{(1 + be^{-ct})^2} \frac{b - e^{ct}}{e^{ct} e^{ct} (1 + be^{-ct})} \end{aligned}$$

Given1 an inflection point of the primitive function $f(t)$ exists, its second order derivative $f''(t)$ must be 0.
 Given2 both $(1 + be^{-ct})^2 e^{ct}$ and $e^{ct} (1 + be^{-ct})$ are denominators that cannot be 0; also, a, b, and c are none zero constants. Therefore, there must be a 0 on $b - e^{ct}$, namely, $t = \frac{\ln b}{c}$

2.3 Estimating model parameters

Model parameters were estimated by using the solver in Microsoft Excel[3,4]. The abstract video was provided in reference[4].

Results

We derived the ogive curve $(y(t) = 4139636.37 / (1 + 2060690.85e^{-0.36t}, R^2 > 0.998)$ from 1997 to 2017 for the publications of mobile research[1]. The inflection point is set at 20th year $(= 20 = (\ln(2060690.85) / 0.36) \approx 40.34 > 20)$ (i.e., 2017 similar to the result from the original study[1], but different in model parameters from the original one[1]). The R-square $(= 0.99)$ is higher in this study than the original one $[1] (= 0.987)$.

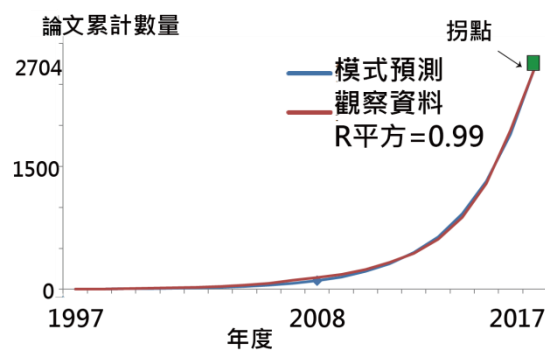


Figure 1 The results compared to the original one[1]

Discussions and conclusion

This provide evident of the determination of inflection point on an ogive curve. The inflection point determined in an scientific way that can be referred to the future relevant studies using the logistic growth curve in prediction and the inflection to verify the trend toward stagnation or the extent of containing the pandemic situation in management[5].

References

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